13. Probability

Question 1.
The variance of random variable X i.e. σ_x^2 or var (X) is equal to
(a) $E(X^2) + [E(X^2)_2]^2$
(b) $E(X) - [E(X^2)]$
(c) $E(X^2) - [E(X)]^2$
(d) None of these
Answer:
(c) $E(X^2) - [E(X)]^2$

Question 2.
A coin is biased so that the head is 3 times likely to occur as a tail. If the coin is tossed twice, then find the probability distribution of the number of tails.

(a)

X	0	1	2
$P(X)$	$\frac{2}{7}$	$\frac{9}{14}$	$\frac{5}{21}$

(b)

X	0	1	2
$P(X)$	$\frac{7}{16}$	$\frac{11}{16}$	$\frac{3}{16}$

(c)

X	0	1	2
$P(X)$	$\frac{3}{16}$	$\frac{9}{21}$	$\frac{1}{9}$

(d)

X	0	1	2
$P(X)$	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

Answer:

(d)

X	0	1	2
$P(X)$	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

Question 3.
A pair of the die is thrown 4 times. If getting a doubled is considered a success, then find the probability distribution of a number of successes.

(a)

X	0	1	2	3	4
$P(X)$	$\frac{625}{1296}$	$\frac{500}{1296}$	$\frac{150}{1296}$	$\frac{20}{1296}$	$\frac{1}{1296}$

(b)

X	0	1	2	3	4
$P(X)$	$\frac{300}{1296}$	$\frac{225}{1296}$	$\frac{425}{1296}$	$\frac{70}{1296}$	$\frac{40}{1296}$

(c)

X	0	1	2	3	4
$P(X)$	$\frac{500}{1296}$	$\frac{700}{1296}$	$\frac{150}{1296}$	$\frac{100}{1296}$	$\frac{3}{1296}$

(d)

X	0	1	2	3	4
$P(X)$	$\frac{375}{1296}$	$\frac{700}{1296}$	$\frac{300}{1296}$	$\frac{30}{1296}$	$\frac{45}{1296}$

Answer:

(a)

X	0	1	2	3	4
$P(X)$	$\frac{625}{1296}$	$\frac{500}{1296}$	$\frac{150}{1296}$	$\frac{20}{1296}$	$\frac{1}{1296}$

Question 4.
Find the probability of throwing atmost 2 sixes in 6 throws of a single die.

(a) $\frac{35}{18}\left(\frac{5}{6}\right)^3$

(b) $\frac{35}{18}\left(\frac{5}{6}\right)^4$

(c) $\frac{18}{29}\left(\frac{2}{3}\right)^4$

(d) $\frac{18}{29}\left(\frac{2}{3}\right)^3$

Answer:

(b) $\frac{35}{18}\left(\frac{5}{6}\right)^4$

Question 5.
A die is thrown again and again until three sixes are obtained. Find the probability of obtaining third six in the sixth throw of the die.

(a) $\frac{625}{23329}$

(b) $\frac{621}{25329}$

(c) $\frac{625}{23328}$
(d) $\frac{620}{23328}$
Answer:
(c) $\frac{625}{23328}$

Question 6.
Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Then, the probability that there is atleast one defective egg is
(a) $1-\frac{7^{10}}{10^{10}}$
(b) $1+\frac{7^{10}}{10^{10}}$
(c) $1+\frac{9^{10}}{10^{10}}$
(d) $1-\frac{9^{10}}{10^{10}}$
Answer:
(d) $1-\frac{9^{10}}{10^{10}}$

Question 7.
The probability of a man hitting a target is $\frac{1}{4}$. How many times must he fire so that the probability of his hitting the target at least once is greater than $\frac{2}{3}$?
(a) 4
(b) 3
(c) 2
(d) 1
Answer:
(a) 4

Question 8.
Eight coins are thrown simultaneously. Find the probability of getting atleast 6 heads.
(a) $\frac{31}{128}$
(b) $\frac{37}{256}$
(c) $\frac{37}{128}$
(d) $\frac{31}{256}$
Answer:
(b) $\frac{37}{256}$

Question 9.
A bag contains 6 red, 4 blue and 2 yellow balls. Three balls are drawn one by one with replacement. Find the probability of getting exactly one red ball.
(a) $\frac{1}{4}$
(b) $\frac{3}{8}$

(c) $\frac{3}{4}$
(d) $\frac{1}{2}$
Answer:
(b) $\frac{3}{8}$

Question 10.
Eight coins are thrown simultaneously. What is the probability of getting atleast 3 heads?
(a) $\frac{37}{246}$
(b) $\frac{21}{256}$
(c) $\frac{219}{256}$
(d) $\frac{19}{246}$
Answer:
(c) $\frac{219}{256}$

Question 11.
For the following probability distribution, the standard deviation of the random variable X is

X	2	3	4
$P(X)$	0.2	0.5	0.3

(a) 0.5
(b) 0.6
(c) 0.61
(d) 0.7
Answer:
(d) 0.7

Question 12.
A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replecement the probability of getting exactly one red ball is
(a) $\frac{45}{196}$
(b) $\frac{135}{392}$
(c) $\frac{15}{56}$
(d) $\frac{15}{29}$
Answer:
(c) $\frac{15}{56}$

Question 13.
A die is thrown and card is selected a random from a deck of 52 playing cards. The probability of gettingan even number on the die and a spade card is
(a) $\frac{1}{2}$

(b) $\frac{1}{4}$
(c) $\frac{1}{8}$
(d) $\frac{3}{4}$
Answer:
(c) $\frac{1}{8}$

Question 14.
A box contains 3 orange balls, 3 green balls and 2 blue balls. Three balls are drawn at random from the box without replacement. The probability of drawing 2 green balls and one blue ball is
(a) $\frac{3}{28}$
(b) $\frac{2}{21}$
(c) $\frac{1}{28}$
(d) $\frac{167}{168}$
Answer:
(a) $\frac{3}{28}$

Question 15.
A flashlight has 8 batteries out of which 3 are dead. If two batteries are selected without replacement and tested, the probability that both are deal is
(a) $\frac{33}{56}$
(b) $\frac{9}{64}$
(c) $\frac{1}{14}$
(d) $\frac{3}{28}$
Answer:
(d) $\frac{3}{28}$

Question 16.
Two dice are thrown. If it is known that the sum of numbers on the dice was less than 6, the probability of getting a sum 3, is
(a) $\frac{1}{18}$
(b) $\frac{5}{18}$
(c) $\frac{1}{5}$
(d) $\frac{2}{5}$
Answer:
(c) $\frac{1}{5}$

Question 17.
Two cards are drawn from a well shuffled deck of 52 playing cards with replacement. The

probability, that both cards are queens, is
(a) $\frac{1}{13} \times \frac{1}{13}$
(b) $\frac{1}{13} + \frac{1}{13}$
(c) $\frac{1}{13} \times \frac{1}{17}$
(d) $\frac{1}{13} \times \frac{4}{51}$
Answer:
(a) $\frac{1}{13} \times \frac{1}{13}$

Question 18.
The probability of guessing correctly at least 8 out of 10 answers on a true-false type examiniation is
(a) $\frac{7}{64}$
(b) $\frac{7}{128}$
(c) $\frac{45}{1024}$
(d) $\frac{7}{41}$
Answer:
(b) $\frac{7}{128}$

Question 19.
The probability distribution of a discrete random variable X is given below:

X	2	3	4	5
$P(X)$	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

The value of k is
(a) 8
(b) 16
(c) 32
(d) 48
Answer:
(c) 32

Question 20.
For the following probability distribution:

X	–4	–3	–2	–1	0
$P(X)$	0.1	0.2	0.3	0.2	0.2

E(X) is equal to
(a) 0
(b) -1
(c) -2

(d) -1.8
Answer:
(d) -1.8

Question 21.
For the following probability distribution

X	1	2	3	4
$P(X)$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$

$E(X^2)$ is equal to
(a) 3
(b) 5
(c) 7
(d) 10
Answer:
(d) 10

Question 22.
Suppose a random variable X follows the binomial distribution with parameters n and p, where $0 < p < 1$. If $p(x = r) / P(x = n - r)$ is dindependent of n and r, then p equals
(a) $\frac{1}{2}$
(b) $\frac{1}{3}$
(c) $\frac{1}{5}$
(d) $\frac{1}{7}$
Answer:
(a) $\frac{1}{2}$

Question 23.
A box has 100 pens of which 10 are defective. What is the probability that out of a sample of 5 pens drawn one by one with replacement at most one is defective?
(a) $\left(\frac{9}{10}\right)^5$
(b) $\frac{1}{2}\left(\frac{9}{10}\right)^4$
(c) $\frac{1}{2}\left(\frac{9}{10}\right)^5$
(d) $\left(\frac{9}{10}\right)^5 + \frac{1}{2}\left(\frac{9}{10}\right)^4$
Answer:
(d) $\left(\frac{9}{10}\right)^5 + \frac{1}{2}\left(\frac{9}{10}\right)^4$

Question 24.
P has 2 children. He has a son, Jatin. What is the probability that Jatin's sibling is a brother?
(a) $\frac{1}{3}$
(b) $\frac{1}{4}$
(c) $\frac{2}{3}$
(d) $\frac{1}{2}$
Answer:
(a) $\frac{1}{3}$

Question 25.
If A and B are 2 events such that P(A) > 0 and P (b) ≠ 1, then $P(\bar{A}/\bar{B}) =$
(a) 1 – P(A|B)
(b) $1 - P(A/\bar{B})$
(c) $\frac{1-P(A \cup B)}{P(B)}$
(d) $\frac{1(\bar{A})}{P(B)}$
Answer:
(b) $1 - P(A/\bar{B})$

Question 26.
If two events A and B area such that $P(\bar{A})$ =0.3, P(B) = 0.4 and $P(B|A \cup \bar{B}) =$
(a) $\frac{1}{2}$
(b) $\frac{1}{3}$
(c) $\frac{2}{5}$
(d) $\frac{1}{4}$
Answer:
(d) $\frac{1}{4}$

Question 27.
If E and F are events such that 0 < P(F) < 1, then
(a) $P(E|F) + P(\bar{E}|F) = 1$
(b) $P(E|F) + P(E|\bar{F}) = 1$
(c) $P(\bar{E}|F) + P(E|\bar{F}) = 1$
(d) $P(E|\bar{F}) + P(\bar{E}|\bar{F}) = 0$
Answer:
(a) $P(E|F) + P(\bar{E}|F) = 1$

Question 28.
P(E ∩ F) is equal to
(a) P(E) . P(F|E)

(b) P(F) . P(E|F)
(c) Both (a) and (b)
(d) None of these
Answer:
(c) Both (a) and (b)

Question 29.
If three events of a sample space are E, F and G, then P(E ∩ F ∩ G) is equal to
(a) P(E) P(F|E) P(G|(E ∩ F))
(b) P(E) P(F|E) P(G|EF)
(c) Both (a) and (b)
(d) None of these
Answer:
(c) Both (a) and (b)

Question 30.
Two cards are drawn at random one by one without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.
(a) $\frac{21}{104}$
(b) $\frac{25}{102}$
(c) $\frac{23}{102}$
(d) $\frac{24}{104}$
Answer:
(b) $\frac{25}{102}$

Question 31.
A bag contains 20 tickets, numbered 1 to 20. A ticket is drawn and then another ticket is drawn without replacement. Find the probability that both tickets will show even numbers.
(a) $\frac{9}{38}$
(b) $\frac{16}{35}$
(c) $\frac{7}{38}$
(d) $\frac{17}{30}$
Answer:
(a) $\frac{9}{38}$

Question 32.
Two balls are drawn one after another (without replacement) from a bag containing 2 white, 3 red and 5 blue balls. What is the probability that atleast one ball is red?
(a) $\frac{7}{15}$
(b) $\frac{8}{15}$
(c) $\frac{7}{16}$

(d) $\frac{5}{16}$
Answer:
(b) $\frac{8}{15}$

Question 33.
Let A and B be independent events with P(A) = 1/4 and P(A ∪ B) = 2P(B) – P(A). Find P(B)
(a) $\frac{1}{4}$
(b) $\frac{3}{5}$
(c) $\frac{2}{3}$
(d) $\frac{2}{5}$
Answer:
(d) $\frac{2}{5}$

Question 34.
Two events A and B will be independent, if
(a) A and B are mutually exclusive
(b) P(A' ∩ B') = [1 – P(A)] [1 – P(B)]
(c) P(A) = P(B)
(d) P(A) + P(B) = 1
Answer:
(c) P(A) = P(B)

Question 35.
If A and B are two independent events such that $P(\bar{A} \cap B) = \frac{2}{15}$ and $P(A \cap \bar{B}) = \frac{1}{6}$, then find P(A) and P (B) respectively.
(a) $\frac{5}{4}, \frac{4}{5}$
(b) $\frac{1}{5}, \frac{1}{7}$
(c) $\frac{1}{6}, \frac{1}{7}$
(d) $\frac{1}{7}, \frac{1}{7}$
Answer:
(a) $\frac{5}{4}, \frac{4}{5}$

Question 36.
If A and B are two independent events, then the probability of occurrence of at least of A and B is given by
(a) 1 – P(A) P(b)
(b) 1 – P(A) P(B')
(c) 1 – P(A') P(B')
(d) 1 – P(A') P(b)

Answer:
(c) 1 – P(A') P(B')

Question 37.
If A and B are two indendent events such that $P(\bar{A})$ = 0.75, P(A ∪ B) = 0.65 and P(b) = P, then find the value of P.
(a) $\frac{9}{14}$
(b) $\frac{7}{15}$
(c) $\frac{5}{14}$
(d) $\frac{8}{15}$
Answer:
(d) $\frac{8}{15}$

Question 38.
If A and Bare events such that P(A) = $\frac{1}{3}$, P(b) = $\frac{1}{4}$ and P(A ∩ B) = $\frac{1}{12}$, then find P(not A and not B).
(a) $\frac{1}{4}$
(b) $\frac{1}{2}$
(c) $\frac{2}{3}$
(d) $\frac{1}{3}$
Answer:
(b) $\frac{1}{2}$

Question 39.
Two cards are drawn successively from a well shuffled pack of 52 cards. Find the probability that one is a red card the other is a queen.
(a) $\frac{103}{1326}$
(b) $\frac{101}{1326}$
(c) $\frac{101}{1426}$
(d) $\frac{103}{1426}$
Answer:
(b) $\frac{101}{1326}$

Question 40.
Given that, the events A and B are such that P(A) = $\frac{1}{2}$, P(A ∪ B) = $\frac{3}{5}$ and P(b) = P. Then probabilities of B if A and B are mutually exclusive and independent respetively are
(a) $\frac{1}{2}$, $\frac{1}{3}$
(b) $\frac{1}{5}$, $\frac{1}{3}$
(c) $\frac{2}{3}$, $\frac{1}{3}$

(d) $\frac{1}{10}, \frac{1}{5}$
Answer:
(d) $\frac{1}{10}, \frac{1}{5}$

Question 41.
Two cards from an ordinary deck of 52 cards are missing. What is the probability that a random card drawn from this deck is a spade?
(a) $\frac{3}{4}$
(b) $\frac{2}{3}$
(c) $\frac{1}{2}$
(d) $\frac{1}{4}$
Answer:
(d) $\frac{1}{4}$

Question 42.
A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.
(a) $\frac{5}{8}$
(b) $\frac{3}{8}$
(c) $\frac{7}{8}$
(d) $\frac{1}{8}$
Answer:
(b) $\frac{3}{8}$

Question 43.
A bag contains 4 balls. Two balls are drawn at random and are found to be white. What is the probability that all balls are white?
(a) $\frac{2}{5}$
(b) $\frac{3}{5}$
(c) $\frac{4}{5}$
(d) $\frac{1}{5}$
Answer:
(b) $\frac{3}{5}$

Question 44.
A bag contains 3 green and 7 white balls. Two balls are drawn one by one at random without replacement. If the second ball drawn is green, what is the probability that the first ball was drawn in also green?
(a) $\frac{5}{9}$

(b) $\frac{4}{9}$
(c) $\frac{2}{9}$
(d) $\frac{8}{9}$
Answer:
(c) $\frac{2}{9}$

Question 45.
A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both clubs. Find the probability of the lost card being a club.
(a) $\frac{11}{50}$
(b) $\frac{17}{50}$
(c) $\frac{13}{50}$
(d) $\frac{19}{50}$
Answer:
(a) $\frac{11}{50}$

Question 46.
A random variable X has the following distribution.

X	1	2	3	4	5	6	7	8
$P(X)$	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the event E = {X is prime number} and F = {X < 4}, P(E ∪ F) =
(a) 0.87
(b) 0.77
(c) 0.35
(d) 0.50
Answer:
(b) 0.77

Question 47.
A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
$P(X)$	a	$4a$	$3a$	$7a$	$8a$	$10a$	$6a$	$9a$

Find P(X < 3), P(X ≥ 4), P(0 < X < 5) respectively.
(a) $\frac{1}{6}, \frac{11}{24}, \frac{33}{48}$
(b) $\frac{1}{6}, \frac{33}{48}, \frac{11}{24}$
(c) $\frac{1}{4}, \frac{11}{26}, \frac{21}{44}$
(d) $\frac{11}{26}, \frac{1}{4}, \frac{21}{44}$
Answer:
(b) $\frac{1}{6}, \frac{33}{48}, \frac{11}{24}$

Question 48.
If the chance that a ship arrives safely at a port is $\frac{9}{10}$; find the chance that out of 5 expected ships, atleast 4 will arrive safely at the port.
(a) $\frac{91854}{100000}$
(b) $\frac{32805}{100000}$
(c) $\frac{59049}{100000}$
(d) $\frac{26244}{100000}$
Answer:
(a) $\frac{91854}{100000}$

Question 49.
If the mean and the variance of a binomial distribution are 4 and, then find $P(X \geq 1)$.
(a) $\frac{720}{729}$
(b) $\frac{721}{729}$
(c) $\frac{728}{729}$
(d) $\frac{724}{729}$
Answer:
(c) $\frac{728}{729}$

Question 50.
A pair of dice is thrown 200 times. If getting a sum of 9 is considered a success, then find the mean and the variance respectively of the number of successes.
(a) $\frac{400}{9}, \frac{1600}{81}$
(b) $\frac{1600}{81}, \frac{400}{9}$
(c) $\frac{1600}{81}, \frac{200}{9}$
(d) $\frac{200}{9}, \frac{1600}{81}$
Answer:
(b) $\frac{1600}{81}, \frac{400}{9}$

Question 51.
In a binomial distribution, the sum of its mean and variance is 1.8. Find the probability of two successes, if the event was conducted times.
(a) 0.2623
(b) 0.2048
(c) 0.302
(d) 0.305
Answer:
(b) 0.2048

Question 52.
If the sum and the product of the mean and variance of a binomial distribution are 24 and 128 respectively, then find the distribution.
(a) $\left(\frac{1}{4}+\frac{3}{4}\right)^{32}$
(b) $\left(\frac{1}{2}+\frac{1}{2}\right)^{30}$
(c) $\left(\frac{1}{2}+\frac{1}{2}\right)^{32}$
(d) $\left(\frac{1}{4}+\frac{3}{4}\right)^{30}$
Answer:
(c) $\left(\frac{1}{2}+\frac{1}{2}\right)^{32}$

Question 53.
If the sum of the mean and variance of a binomial distribution is 15 and the sum of their squares is 17, then find the distribution.
(a) $\left(\frac{2}{3}+\frac{1}{3}\right)^{25}$
(b) $\left(\frac{1}{2}+\frac{1}{2}\right)^{25}$
(c) $\left(\frac{1}{2}+\frac{1}{2}\right)^{27}$
(d) $\left(\frac{2}{3}+\frac{1}{3}\right)^{27}$
Answer:
(d) $\left(\frac{2}{3}+\frac{1}{3}\right)^{27}$

Question 54.
The mean and the variance of a binomial distribution are 4 and 2 respectively. Find the probability of atleast 6 successes.
(a) $\frac{37}{256}$
(b) $\frac{32}{255}$
(c) $\frac{34}{259}$
(d) $\frac{31}{256}$
Answer:
(a) $\frac{37}{256}$

Question 55.
If $P(A \cap B) = \frac{7}{10}$ and $P(b) = \frac{17}{20}$, P(A|B) equals
(a) $\frac{14}{17}$

(b) $\frac{17}{20}$
(c) $\frac{7}{8}$
(d) $\frac{1}{8}$
Answer:
(a) $\frac{14}{17}$

Question 56.
If P(A) = $\frac{3}{10}$, P(b) = $\frac{2}{5}$ and P(A ∪ B) = $\frac{3}{5}$, then P(B|A) + P(A|B) equals
(a) $\frac{1}{4}$
(b) $\frac{1}{3}$
(c) $\frac{5}{12}$
(d) $\frac{7}{12}$
Answer:
(d) $\frac{7}{12}$

Question 57.
If P(A) = $\frac{2}{5}$, P(B) = $\frac{3}{10}$ and P(A ∩ B) = $\frac{1}{5}$, then P(A'|B') . (P(B'|A') is equal to
(a) $\frac{5}{6}$
(b) $\frac{5}{7}$
(c) $\frac{25}{42}$
(d) 1
Answer:
(b) $\frac{5}{7}$

Question 58.
If A and B are two events sue that P(A) = $\frac{1}{2}$, P(b) = $\frac{1}{3}$, P(A|B) = $\frac{1}{4}$ then (A' ∩ B') equals
(a) $\frac{1}{12}$
(b) $\frac{3}{4}$
(c) $\frac{1}{4}$
(d) $\frac{3}{16}$
Answer:
(c) $\frac{1}{4}$

Question 59.
If P(A) = 0.4, P(b) = 0.8 and P(B|A) = 0.6, then P(A ∪ B) equal to
(a) 0.24
(b) 0.3
(c) 0.48
(d) 0.96

Answer:
(c) 0.48

Question 60.
If A and B are two events and A ≠ Φ, B ≠ Φ, then
(a) P(A|B) = P(A) . P(b)
(b) P(A|B) = $\frac{P(A \cap B)}{P(B)}$
(c) P(A|B) . P(B|A) = 1
(d) P(A|B) = P(A)|P(b)
Answer:
(b) P(A|B) = $\frac{P(A \cap B)}{P(B)}$

Question 61.
A and B are events such that P(A) = 0.4, P(b) = 0.3 and P(A ∪ B) = 0.5. Then P(B' ∩ A) equals
(a) $\frac{2}{3}$
(b) $\frac{1}{2}$
(c) $\frac{3}{10}$
(d) $\frac{1}{5}$
Answer:
(d) $\frac{1}{5}$

Question 62.
You are given that A and B are two events such that P(b) = $\frac{3}{5}$, P(A|B) = = $\frac{4}{5}$, then P(A) equals
(a) $\frac{3}{10}$
(b) $\frac{1}{5}$
(c) $\frac{1}{2}$
(d) $\frac{3}{5}$
Answer:
(c) $\frac{1}{2}$